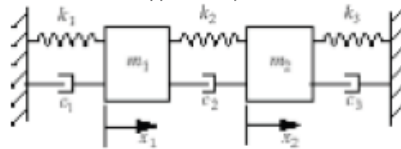


Chapter 5

Review and HW assignment

Example 1, Chapter 5

Consider the system of **the figure** For $c_1 = c_2 = c_3 = 0$, derive the equation of motion and calculate the mass and stiffness matrices. Note that setting $k_3 = 0$ in your solution should result in the stiffness matrix given by



Solution:

For mass 1:

$$m_1 \ddot{x}_1 = -k_1 x_1 + k_2 (x_2 - x_1)$$

$$\Rightarrow m_1 \ddot{x}_1 + (k_1 + k_2) x_1 - k_2 x_2 = 0$$

For mass 2:

$$m_2 \ddot{x}_2 = -k_3 x_2 - k_2 (x_2 - x_1)$$

$$\Rightarrow m_2 \ddot{x}_2 - k_2 x_1 + (k_2 + k_3) x_2 = 0$$

So, $M\ddot{\mathbf{x}} + K\mathbf{x} = 0$ Thus:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \ddot{\mathbf{x}} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix} \mathbf{x} = 0$$

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \quad K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix}$$

Calculate the characteristic equation from **the above system**

$m_1 = 9 \text{ kg}$ $m_2 = 1 \text{ kg}$ $k_1 = 24 \text{ N/m}$ $k_2 = 3 \text{ N/m}$ $k_3 = 3 \text{ N/m}$ and solve for the system's natural frequencies.

Solution: Characteristic equation is found from

$$\det(-\omega^2 M + K) = 0$$

$$\begin{vmatrix} -\omega^2 m_1 + k_1 + k_2 & -k_2 \\ -k_2 & -\omega^2 m_2 + k_2 + k_3 \end{vmatrix} = \begin{vmatrix} -9\omega^2 + 27 & -3 \\ -3 & -\omega^2 + 6 \end{vmatrix} = 0$$

$$9\omega^4 - 81\omega^2 + 153 = 0$$

Solving for ω :

$$\omega_1 = 1.642 \text{ rad/s}$$

$$\omega_2 = 2.511 \text{ rad/s}$$

Calculate the vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ for **the above system**

Solution: Calculate $\mathbf{u}_1$:

eigenvectors

$$\begin{bmatrix} (-2.697)(9) + 27 & -3 \\ -3 & -2.697 + 6 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{This yields}$$

$$2.727u_{11} - 3u_{21} = 0$$

$$-3u_{11} + 3.303u_{21} = 0 \quad \text{or, } u_{21} = 0.909u_{11}$$

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0.909 \end{bmatrix}$$

Calculate $\mathbf{u}_2$:

$$\begin{bmatrix} (-6.303)(9) + 27 & -3 \\ -3 & -6.303 + 6 \end{bmatrix} \begin{bmatrix} u_{12} \\ u_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{This yields}$$

$$-29.727u_{12} - 3u_{22} = 0$$

$$-3u_{12} + 0.303u_{22} = 0 \quad \text{or, } u_{12} = -0.101u_{22}$$

$$\mathbf{u}_2 = \begin{bmatrix} -0.101 \\ 1 \end{bmatrix}$$

For initial conditions $\mathbf{x}(0) = [1 \ 0]^T$ and $\dot{\mathbf{x}}(0) = [0 \ 0]^T$ calculate the free response of the **given in problem of Example 1** and Plot the response x_1 and x_2 .

Solution: Given $\mathbf{x}(0) = [1 \ 0]^T$, $\dot{\mathbf{x}}(0) = [0 \ 0]^T$, The solution is

$$\mathbf{x}(t) = A_1 \sin(\omega_1 t + \phi_1) \mathbf{u}_1 + A_2 \sin(\omega_2 t + \phi_2) \mathbf{u}_2$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} A_1 \sin(\omega_1 t + \phi_1) - 0.101 A_2 \sin(\omega_2 t + \phi_2) \\ 0.909 A_1 \sin(\omega_1 t + \phi_1) + A_2 \sin(\omega_2 t + \phi_2) \end{bmatrix}$$



$$1 = A_1 \sin \phi_1 - 0.101 A_2 \sin \phi_2 \quad [1]$$

$$0 = 0.909 A_1 \sin \phi_1 + A_2 \sin \phi_2 \quad [2]$$

$$0 = 1.642 A_1 \cos \phi_1 - 0.2536 A_2 \cos \phi_2 \quad [3]$$

$$0 = 6.033 A_1 \cos \phi_1 + 2.511 A_2 \cos \phi_2 \quad [4]$$

Using initial conditions,

From [3] and [4], $\phi_1 = \phi_2 = \pi / 2$

From [1] and [2], $A_1 = 0.916$, and $A_2 = -0.833$

So,

$$x_1(t) = 0.916 \sin(1.642t + \pi / 2) + 0.0841 \sin(2.511t + \pi / 2)$$

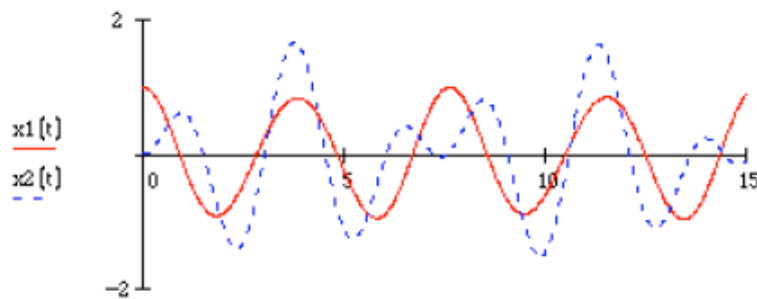
$$x_2(t) = 0.833 \sin(1.642t + \pi / 2) - 0.833 \sin(2.511t + \pi / 2)$$

$$\mathbf{x}_1(t) = 0.916 \cos 1.642t + 0.0841 \cos 2.511t$$

$$\mathbf{x}_2(t) = 0.833 (\cos 1.642t - \cos 2.511t)$$

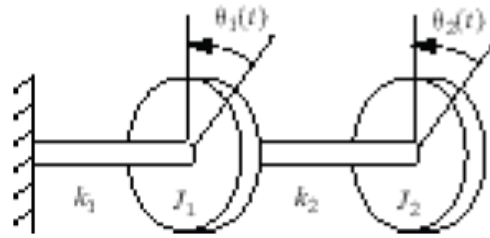
$$x1(t) := 0.916 \cdot \cos(1.642 \cdot t) + 0.0841 \cdot \cos(2.511 \cdot t)$$

$$x2(t) := 0.833 \cdot (\cos(1.642 \cdot t) - \cos(2.511 \cdot t))$$



Example 2, Chapter 5

Determine the equation of motion in matrix form, then calculate the natural frequencies and mode shapes of the torsional system of Figure shown. Assume that the torsional stiffness values provided by the shaft are equal ($k_1 = k_2$) and that disk 1 has three times the inertia as that of disk 2 ($J_1 = 3J_2$).



Solution: Let $k = k_1 = k_2$ and $J_1 = 3J_2$. The equations of motion are

$$\text{So, } J_2 \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \ddot{\theta} + k \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \theta = 0$$

$$\begin{aligned} J_1 \ddot{\theta}_1 + 2k\theta_1 - k\theta_2 &= 0 \\ J_2 \ddot{\theta}_2 - k\theta_1 + k\theta_2 &= 0 \end{aligned}$$

Calculate the natural frequencies:

$$\det(-\omega^2 J + K) = \begin{vmatrix} -3\omega^2 J_2 + 2k & -k \\ -k & -\omega^2 J_2 + k \end{vmatrix} = 0$$

$$\omega_1 = 0.482 \sqrt{\frac{k}{J_2}} \quad \omega_2 = 1.198 \sqrt{\frac{k}{J_2}}$$

Calculate the mode shapes: mode shape 1:

$$\text{So, } \mathbf{u}_1 = \begin{bmatrix} 0.7676 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -3(0.2324)k + 2k & -k \\ -k & -(0.2324)k + k \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} = 0$$

$$u_{11} = 0.7676 u_{12}$$

mode shape 2:

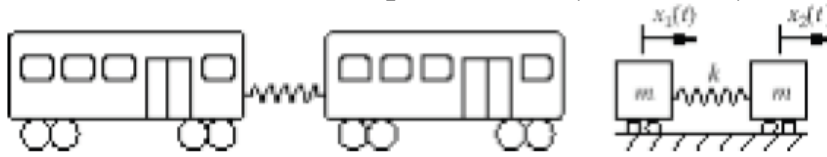
$$\begin{bmatrix} -3(1.434)k + 2k & -k \\ -k & -(1.434)k + k \end{bmatrix} \begin{bmatrix} u_{21} \\ u_{22} \end{bmatrix} = 0$$

$$\text{So, } \mathbf{u}_2 = \begin{bmatrix} -0.434 \\ 1 \end{bmatrix}$$

$$u_{21} = -0.434 u_{22}$$

Example 3, Chapter 5

Two subway cars of Fig. shown have 2000 kg mass each and are connected by a coupler. The coupler can be modeled as a spring of stiffness $k = 280,000$ N/m. Write the equation of motion and calculate the natural frequencies and (normalized) mode shapes.



Solution: Given: $m_1 = m_2 = m = 2000$ kg $k = 280,000$ N/m

The equations of motion are: $m\ddot{x}_1 + kx_1 - kx_2 = 0$

$$m\ddot{x}_2 - kx_1 + kx_2 = 0$$

In matrix form this becomes:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \ddot{\mathbf{x}} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \mathbf{x} = 0$$

$$\begin{bmatrix} 2000 & 0 \\ 0 & 2000 \end{bmatrix} \ddot{\mathbf{x}} + \begin{bmatrix} 280,000 & -280,000 \\ -280,000 & 280,000 \end{bmatrix} \mathbf{x} = 0$$

Natural frequencies:

$$\det(-\omega^2 M + K) = 0$$

$$\begin{vmatrix} -2000\omega^2 + 280,000 & -280,000 \\ -280,000 & -2000\omega^2 + 280,000 \end{vmatrix} = 0$$

$$4 \times 10^6 \omega^4 - 1.12 \times 10^9 \omega^2 = 0$$

$$\omega^2 = 0, 280 \Rightarrow \omega_1 = 0 \text{ rad/sec and } \omega_2 = 16.73 \text{ rad/sec}$$

Mode shapes:

Mode 1, $\omega_1^2 = 0$ $\begin{bmatrix} 280,000 & -280,000 \\ -280,000 & 280,000 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \therefore u_{11} = u_{12}$

$\begin{bmatrix} -280,000 & -280,000 \\ -280,000 & -280,000 \end{bmatrix} \begin{bmatrix} u_{21} \\ u_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \therefore u_{21} = u_{22} \quad \mathbf{u}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Normalizing the mode shapes yields

$$\mathbf{u}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{u}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Note that $\mathbf{u}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is also acceptable because a mode shape times a constant (-1 in this case) is still a mode shape.

Example 4, Chapter 5

Suppose that the subway cars of the above problem given the initial position of $x_{10} = 0$, $x_{20} = 0.1$ m and initial velocities of $v_{10} = v_{20} = 0$. Calculate the response of the cars.

Solution:

$$\text{Given: } \mathbf{x}(0) = \begin{bmatrix} 0 & 0.1 \end{bmatrix}^T, \dot{\mathbf{x}}(0) = \mathbf{0}$$

$$\text{From problem } , \quad \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } \mathbf{u}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$
$$\omega_1 = 0 \text{ rad/s and } \omega_2 = 16.73 \text{ rad/s}$$

The solution is

$$\mathbf{x}(t) = (c_1 + c_2 t) \mathbf{u}_1 + A \sin(16.73t + \phi) \mathbf{u}_2$$
$$\Rightarrow \dot{\mathbf{x}}(0) = c_2 \mathbf{u}_1 + 16.73A \cos(\phi) \mathbf{u}_2 \text{ and } \mathbf{x}(0) = c_1 \mathbf{u}_1 + A \sin(\phi) \mathbf{u}_2$$

Using initial the conditions four equations in four unknowns result:

$$\begin{aligned} 0 &= c_1 + A \sin \phi & [1] \\ 0.1 &= c_1 - A \sin \phi & [2] \\ 0 &= c_2 + 16.73A \cos \phi & [3] \\ 0 &= c_2 - 16.73A \cos \phi & [4] \end{aligned}$$

$$\text{From [3] and [4]: } c_2 = 0, \text{ and } \phi = \frac{\pi}{2} \text{ rad}$$

$$\text{From [1] and [2]: } c_1 = 0.05 \text{ m and } A = -0.05 \text{ m}$$

The solution is

$$x_1(t) = 0.05 - 0.05 \cos 16.73t$$
$$x_2(t) = 0.05 + 0.05 \cos 16.73t$$

Note that if $\mathbf{u}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is chosen as the second mode shape the answer will remain the same. It might be worth presenting both solutions in class, as students are often skeptical that the two choices will yield the same result.

Example 5, Chapter 5

Compute the natural frequencies of the following system:

$$\begin{bmatrix} 6 & 2 \\ 2 & 4 \end{bmatrix} \ddot{\mathbf{x}}(t) + \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} \mathbf{x}(t) = 0.$$

Solution:

$$\det(-\omega^2 M + K) = \det\left(-\omega^2 \begin{bmatrix} 6 & 2 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}\right) = 20\omega^4 - 22\omega^2 + 2 = 0, \omega^2 = 0.1, 1$$

$$\underline{\omega_{1,2} = 0.316, 1 \text{ rad/s}}$$

Example 6, Chapter 5

A slightly more sophisticated model of a vehicle suspension system is given

Write the equations of motion in matrix form. Calculate the natural frequencies for $k_1 = 10^3$ N/m, $k_2 = 10^4$ N/m, $m_2 = 50$ kg, and $m_1 = 2000$ kg.

Solution: The equations of motion are

$$2000\ddot{x}_1 + 1000x_1 - 1000x_2 = 0$$

$$50\ddot{x}_2 - 1000x_1 + 11,000x_2 = 0$$

In matrix form this becomes:

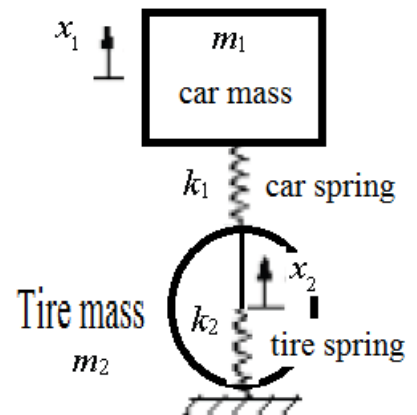
$$\begin{bmatrix} 2000 & 0 \\ 0 & 50 \end{bmatrix} \ddot{\mathbf{x}} + \begin{bmatrix} 1000 & -1000 \\ -1000 & 11,000 \end{bmatrix} \mathbf{x} = 0$$

Natural frequencies:

$$\det(-\omega^2 M + K) = 0$$

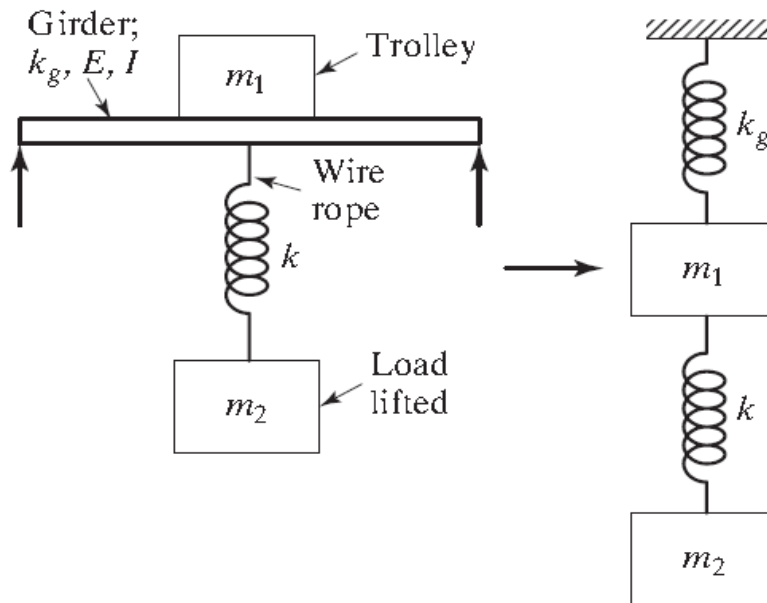
$$\begin{vmatrix} -2000\omega^2 + 1000 & -1000 \\ -1000 & -50\omega^2 + 11,000 \end{vmatrix} = 100,000\omega^4 - 2.205 \times 10^7 \omega^2 + 10^7 = 0$$

$$\omega_{1,2}^2 = 0.454, 220.046 \Rightarrow \omega_1 = 0.674 \text{ rad/s} \text{ and } \omega_2 = 14.8 \text{ rad/s}$$



Example 7, Chapter 5

An electric overhead traveling crane, consisting of a girder, trolley, and wire rope, is shown in Figure . The girder has a flexural rigidity (EI) of 6×10^{12} lb-in.² and a span (L) of 30 ft. The rope is made of steel and has a length (ℓ) of 20 ft. The weights of the trolley and the load lifted are 8000 lb and 2000 lb, respectively. Find the area of cross section of the rope such that the fundamental natural frequency is greater than 20 Hz.



$$k_g = \text{stiffness of girder} = \frac{48 EI}{\ell^3} = \frac{48 (6 (10^{12}))}{(30 (12))^3} = 6.1728 (10^6) \text{ lb/in}$$

$$k = \text{stiffness of rope} = \frac{AE}{\ell} = \frac{A (30 (10^6))}{(20) (12)} = 12.5 (10^4) A \text{ lb/in}$$

$$m_1 = \text{mass of trolley} = 8000/386.4 = 20.7039 \text{ lb-sec}^2/\text{in}$$

$$m_2 = \text{mass of load} = 2000/386.4 = 5.1760 \text{ lb-sec}^2/\text{in}$$

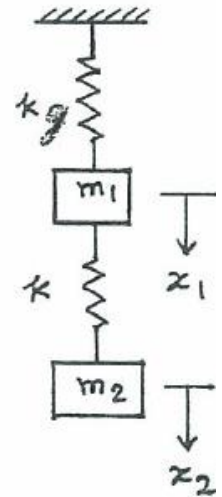
Desired frequency value: $\omega_1 > 20$ Hz. Let $\omega_1 = 25$ Hz = 157.08 rad/sec or $\omega_1^2 = 24674.1264$ (rad/sec)².

Fundamental natural frequency is given by (see Eq. (E3) in solution of Problem 5.5):

$$\omega_1^2 = \frac{k_g + k}{2 m_1} + \frac{k}{2 m_2} - \sqrt{\frac{1}{4} \left(\frac{k_g + k}{m_1} + \frac{k}{m_2} \right)^2 - \frac{k_g k}{m_1 m_2}} \quad (1)$$

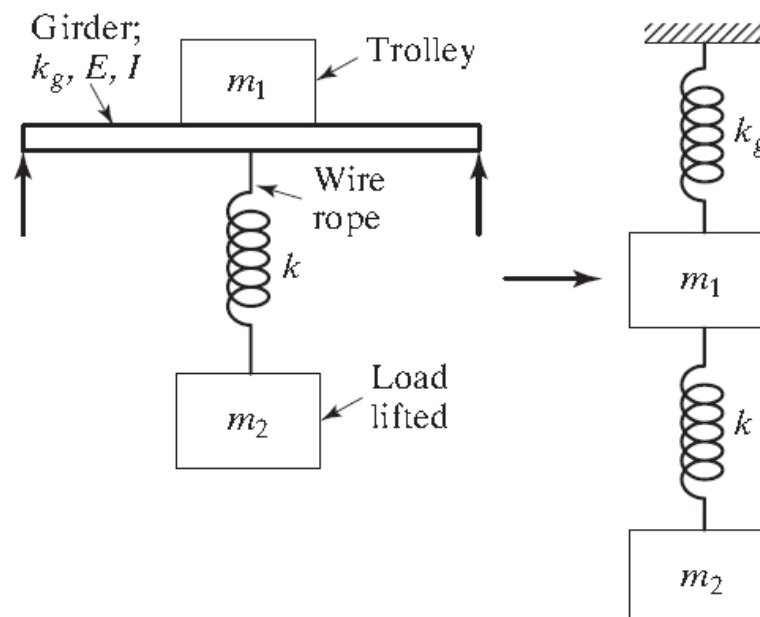
Using the known values of k_g , m_1 , and m_2 , a series of trial values of A are given and Eq. (1) is evaluated to find the corresponding values of ω_1^2 . The results are given in the table below. It can be seen that $A = 1.1$ in² yields a frequency that satisfies the specification.

A (in ²)	k (lb/in)	ω_1^2 (rad/sec) ²	ω_1 (rad/sec)
0.6	0.7500 (10 ⁵)	0.1434 (10 ⁵)	119.7
0.7	0.8750 (10 ⁵)	0.1715 (10 ⁵)	131.0
0.8	0.1000 (10 ⁶)	0.1946 (10 ⁵)	139.5
0.9	0.1125 (10 ⁶)	0.2176 (10 ⁵)	147.5
1.0	0.1250 (10 ⁶)	0.2406 (10 ⁵)	155.1
1.1	0.1375 (10 ⁶)	0.2662 (10 ⁵)	163.2
1.2	0.1500 (10 ⁶)	0.2918 (10 ⁵)	170.8
1.3	0.1625 (10 ⁶)	0.3123 (10 ⁵)	176.7
1.4	0.1750 (10 ⁶)	0.3379 (10 ⁵)	183.8
1.5	0.1875 (10 ⁶)	0.3610 (10 ⁵)	190.0



Example 8, Chapter 5

An overhead traveling crane can be modeled as indicated in Figure . Assuming that the girder has a span of 40 m, an area moment of inertia (I) of 0.02 m⁴, and a modulus of elasticity (E) of 2.06×10^{11} N/m², the trolley has a mass (m_1) of 1000 kg, the load being lifted has a mass (m_2) of 5000 kg, and the cable through which the mass (m_2) is lifted has a stiffness (k) of 3.0×10^5 N/m, determine the natural frequencies and mode shapes of the system.



$$k_1 = \frac{48EI}{l^3} = \frac{48(2.06 \times 10^{11})(0.02)}{(40)^3} = 3.09 \times 10^6 \text{ N/m}$$

$$k_2 = 0.3 \times 10^6 \text{ N/m}, \quad m_1 = 1000 \text{ kg}, \quad m_2 = 5000 \text{ kg}$$

$E_3, (E_3)$ in the solution of problem 5.5 gives

$$\omega_1^2, \omega_2^2 = \frac{3.39 \times 10^6}{2000} + \frac{0.3 \times 10^6}{10000} \mp \sqrt{\frac{1}{4} \left(\frac{3.39 \times 10^6}{1000} + \frac{0.3 \times 10^6}{5000} \right)^2 - \frac{3.09 \times 0.3 \times 10^{12}}{5 \times 10^6}}$$

$$= (1.725 \mp 1.6704) \times 10^3$$

$$\omega_1 = 7.3892 \text{ rad/s}, \quad \omega_2 = 58.2701 \text{ rad/s}$$

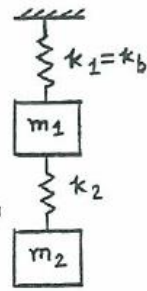
From $E_4, (E_4)$ and (E_5) of solution of problem 5.5,

$$r_1 = \frac{k_2}{-m_2 \omega_1^2 + k_2} = \frac{0.3 \times 10^6}{-5000(54.6003) + 0.3 \times 10^6} = 11.1117$$

$$r_2 = \frac{k_2}{-m_2 \omega_2^2 + k_2} = \frac{0.3 \times 10^6}{-5000(3395.4046) + 0.3 \times 10^6} = -0.01799$$

Mode shapes are

$$\vec{X}^{(1)} = \begin{Bmatrix} 1.0 \\ 11.1117 \end{Bmatrix} X_1^{(1)}, \quad \vec{X}^{(2)} = \begin{Bmatrix} 1.0 \\ -0.01799 \end{Bmatrix} X_1^{(2)}$$

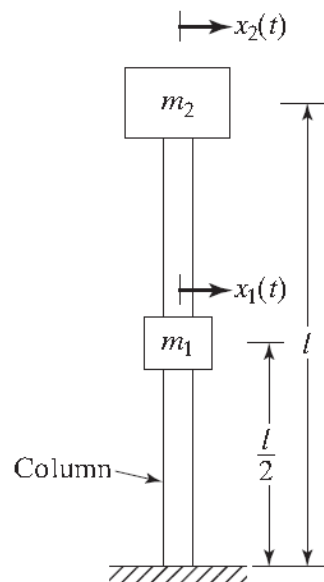
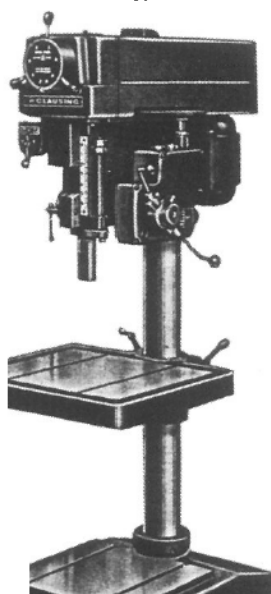


Example 9, Chapter 5

The drilling machine shown in the Figure can be modeled as a two-degree-of-freedom system as indicated in the Figure. Since a transverse force applied to mass m_1 or mass m_2 causes both the masses to deflect, the system exhibits elastic coupling. The bending stiffnesses of the column are given by (see Section 6.4 for the definition of stiffness influence coefficients)

$$k_{11} = \frac{768 EI}{7 l^3}, \quad k_{12} = k_{21} = -\frac{240 EI}{7 l^3}, \quad k_{22} = \frac{96 EI}{7 l^3}$$

Determine the natural frequencies of the drilling machine.



Frequency equation:

$$\left| \left[-\omega^2 [m] + [k] \right] \right| = 0$$

or

$$\begin{vmatrix} (k_{11} - \omega^2 m_1) & k_{12} \\ k_{21} & (k_{22} - \omega^2 m_2) \end{vmatrix} = 0 \quad (1)$$

Expansion of the determinantal equation (1) gives:

$$(m_1 m_2) \omega^4 - (m_1 k_{22} + m_2 k_{11}) \omega^2 + (k_{11} k_{22} - k_{12}^2) = 0 \quad (2)$$

Roots of Eq. (2):

$$\omega_2^2, \omega_1^2 = \frac{(m_1 k_{22} + m_2 k_{11}) \pm \sqrt{(m_1 k_{22} - m_2 k_{11})^2 + 4 m_1 m_2 k_{12}^2}}{2 m_1 m_2} \quad (3)$$

Substitution of known expressions for k_{11} , k_{12} , and k_{22} into Eq. (3) yields:

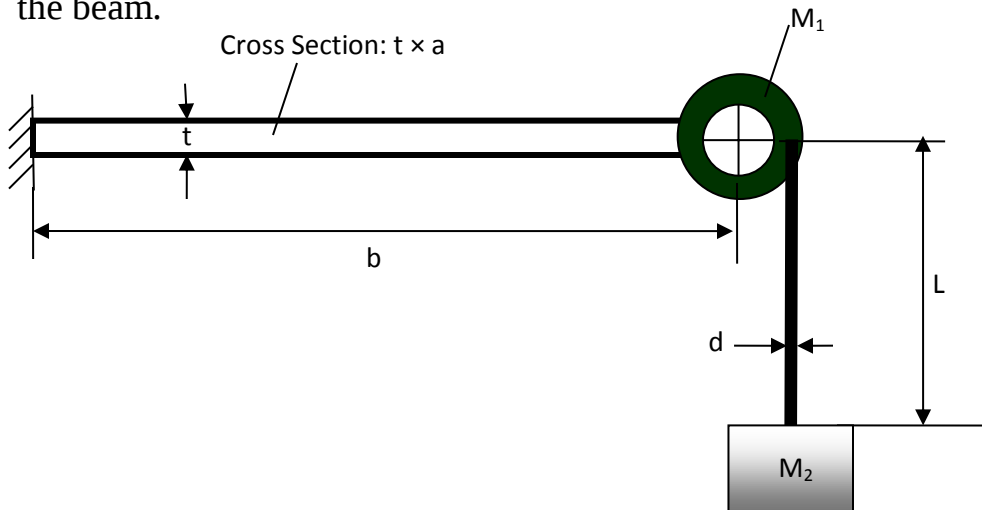
$$\omega_2^2, \omega_1^2 = \frac{48}{7} \frac{E I}{m_1 m_2} \left[(m_1 + 8 m_2) \pm \sqrt{(m_1 - 8 m_2)^2 + 25 m_1 m_2} \right] \quad (4)$$

Exercises

Solve the following problems as HW

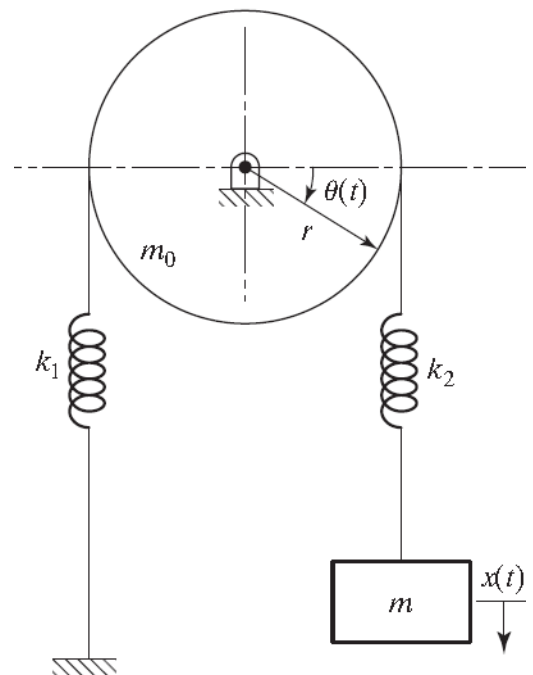
Exercise 1 A hosting drum; having a mass M_1 , is mounted at the end of a steel cantilever beam of thickness t , width a , and length b , as shown. The wire rope is made of steel and it has a diameter of d and length L . If the mass hanged at the end of the rope is M_2 . In terms of M_1 , t , a , b , d , L , M_2 and E (Modulus of elasticity of steel):

- Write the equations of motion for the system, and
- Derive expressions for the natural frequencies of the system.
- Now, design the Cantilever beam, in order to have the natural frequencies of the system greater than 4 Hz, when $M_2 = 40$ (kg), $M_1 = 20$ (kg), $t = 0.06$ (m), $b = 2$ (m), $d = 0.06$ (m), $L = 1$ (m) and E (Modulus of elasticity of steel) = 200 (G Pas.), that is, find the width of the beam.

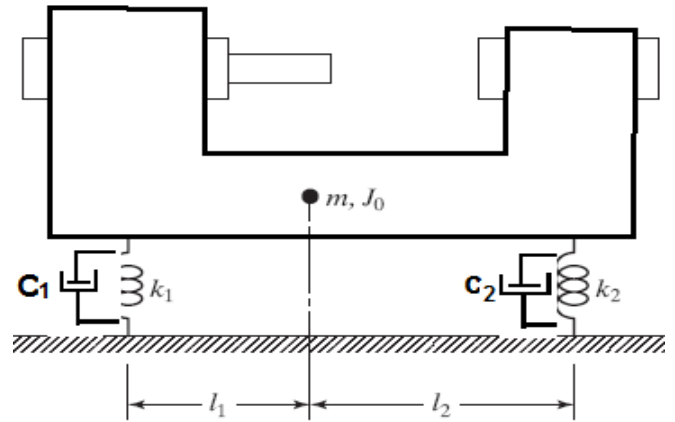


Exercise 2

Determine the natural frequencies of the problem shown assuming that the rope passes over the pulley and does not slip.

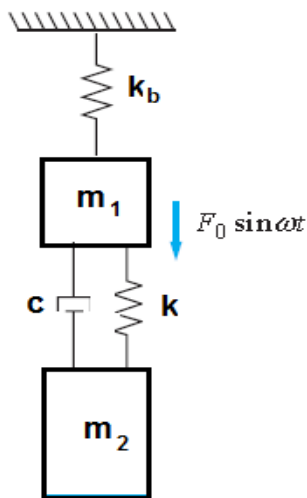


Exercise 3 A machine tool (lathe machine) having a mass of $m = 1200 \text{ kg}$ and a mass moment of inertia $J_o = 340 \text{ kg} - \text{m}^2$ is supported on a viscoelastic supports as shown in Fig.1, if the stiffness at the supports are given by $k_1 = 3500 \text{ N/mm}$ and $k_2 = 2500 \text{ N/mm}$, the damping constants of the supports are given by $c_1 = 550 \text{ N-s/mm}$ and $c_2 = 480 \text{ N-s/mm}$. The supports are located at $l_1 = 600 \text{ mm}$ and $l_2 = 800 \text{ mm}$ from the center of gravity (C.G.).



- For this two degree of freedom system write the equation of motion in terms vertical displacement of the C.G and the angular displacement about the C.G.
- Find the natural frequencies after neglecting the damping effect of the supports.
- Find the mode shapes of the machine tool when the damping effect is neglected.
- Find the general solution when the damping effect is neglected.

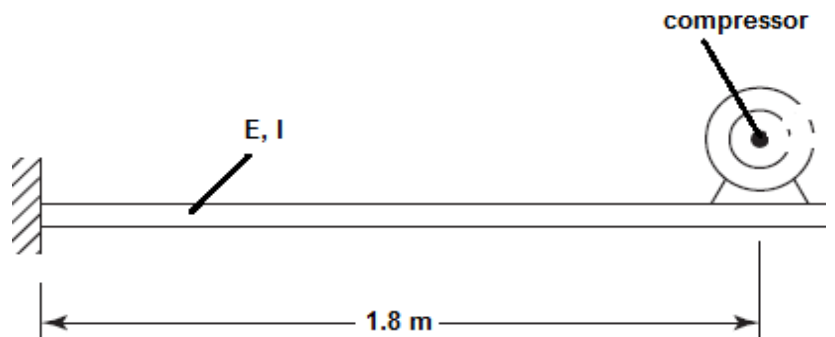
Exercise 4 Use the Laplace transform method to analyze the situation of a damped absorber (k, c) attached to un-damped system shown. Determine the steady state response of mass m_1, m_2 (do not take the inverse of Laplace Transform and assume zero initial conditions).



- If the model given in part (A) is a model of a $m_1 = 300 \text{ kg}$ compressor which is placed at the end of a massless-cantilever beam of length 1.8 m , elastic modulus $E = 200 \text{ GPa}$, and moment of inertia $I = 0.000018 \text{ m}^4$. When the compressor operates (without absorber, that is $c = 0$ and $k = 0$) at 1000 rpm , it has a steady state amplitude of 1.2 mm , Find the force amplitude (F_0).

- What is the compressor's steady state amplitude, if a 30 kg absorber is added at the end of the beam, the absorber is of damping coefficient $c = 500 \text{ N-s/m}$ and stiffness $k = 130000 \text{ N/m}$,

- Discuss your results



Exercise 5

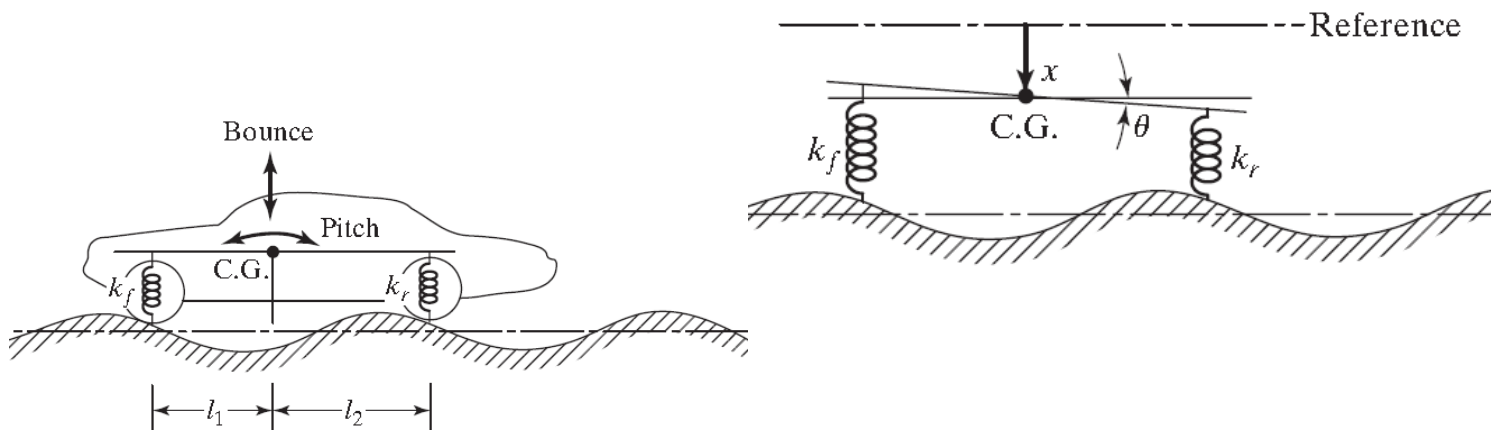
The mass and stiffness matrices and the mode shapes of a two-degree-of-freedom system are given by

$$[m] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, \quad [k] = \begin{bmatrix} 27 & -3 \\ -3 & 3 \end{bmatrix}, \quad \bar{X}^{(1)} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}, \quad \bar{X}^{(2)} = \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$$

If the first natural frequency is given by $\omega_1 = 1.4142$, determine the masses m_1 and m_2 and the second natural frequency of the system.

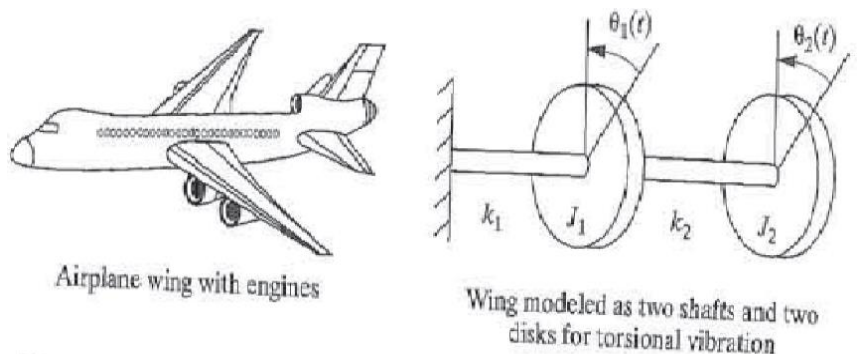
Exercise 6

An automobile is modeled with a capability of pitch and bounce motions, as shown in Fig. 5.45. It travels on a rough road whose surface varies sinusoidally with an amplitude of 0.05 m and a wavelength of 10 m. Derive the equations of motion of the automobile for the following data: mass = 1,000 kg, radius of gyration = 0.9 m, $l_1 = 1.0$ m, $l_2 = 1.5$ m, $k_f = 18$ kN/m, $k_r = 22$ kN/m, velocity = 50 km/hr.



Exercise 7

The torsional vibration of wing of an airplane is modeled as shown in the following Figure, derive the equations of motion then write them in matrix form and calculate the natural frequencies in terms of rotational inertia and stiffness and of wing.

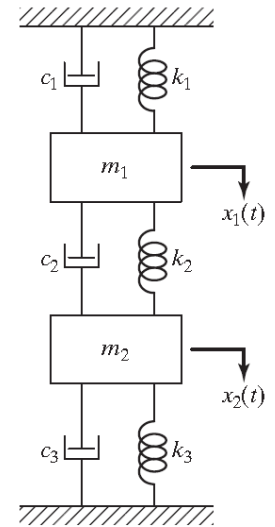
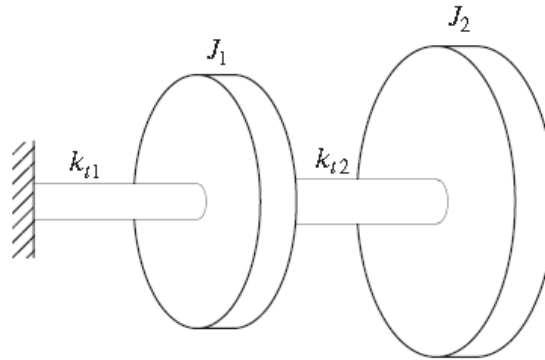


Exercise 8

Find the displacements $x_1(t)$ and $x_2(t)$ in the Fig. for $m_1 = 1$ kg, $m_2 = 2$ kg, $k_1 = k_2 = k_3 = 10,000$ N/m, and $c_1 = c_2 = c_3 = 2000$ N-s/m using the initial conditions $x_1(0) = 0.2$ m, $x_2(0) = 0.1$ m, and $\dot{x}_1(0) = \dot{x}_2(0) = 0$.

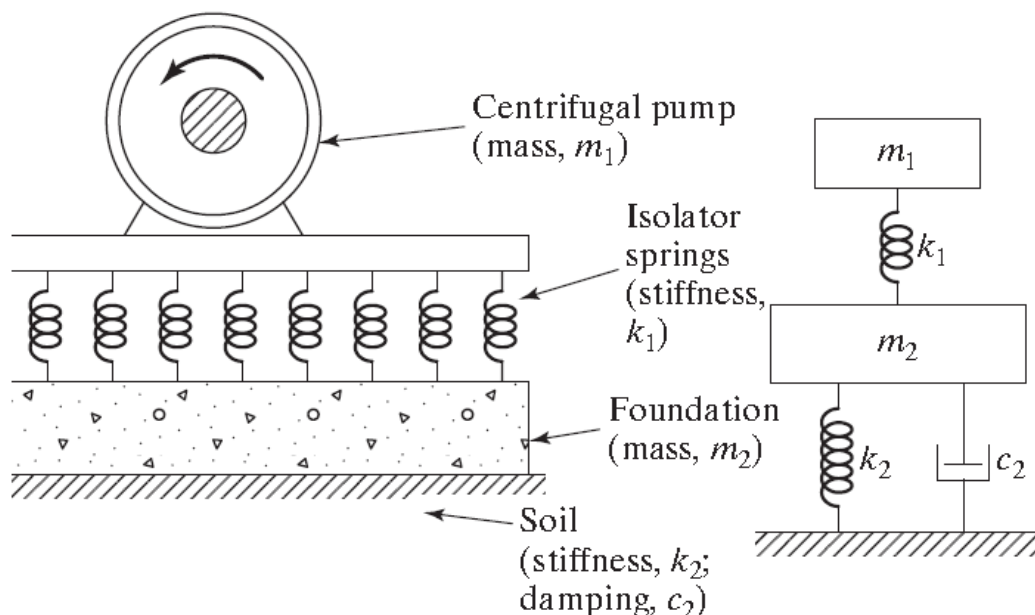
Exercise 9

Determine the natural frequencies of the problem shown assuming $k_{t2} = 2k_{t1}$ and $J_2 = J_1$



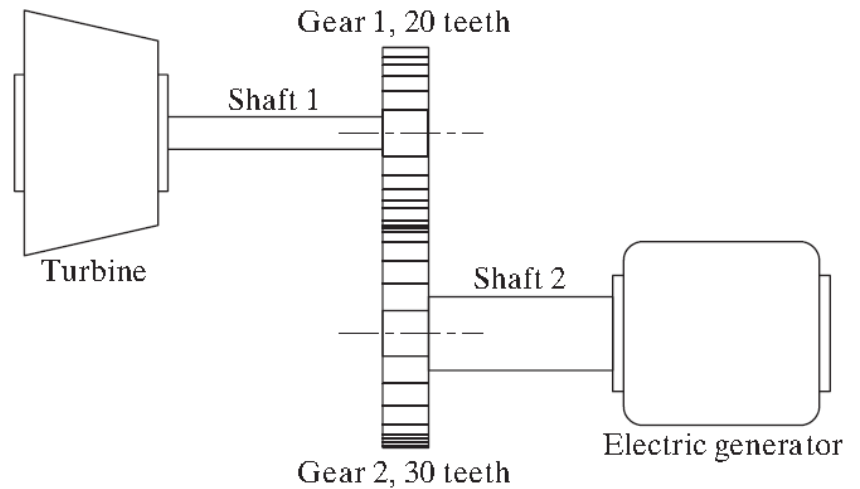
Exercise 10

A centrifugal pump, having an unbalance of me , is supported on a rigid foundation of mass m_2 through isolator springs of stiffness k_1 , as shown in Fig. 5.54. If the soil stiffness and damping are k_2 and c_2 , find the displacements of the pump and the foundation for the following data: $mg = 0.5$ lb, $e = 6$ in., $m_1g = 800$ lb, $k_1 = 2000$ lb/in., $m_2g = 2000$ lb, $k_2 = 1000$ lb/in., $c_2 = 200$ lb-sec/in., and speed of pump = 1200 rpm.



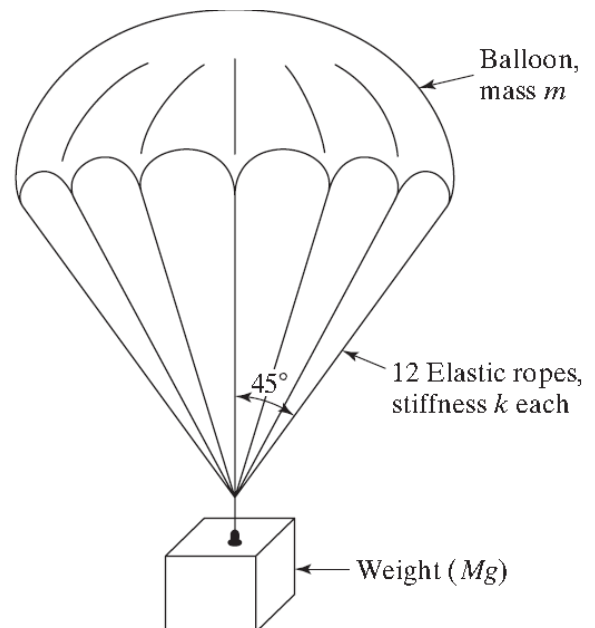
Exercise 11

A turbine is connected to an electric generator through gears, as shown in the Fig. . The mass moments of inertia of the turbine, generator, gear 1, and gear 2 are given, respectively, by 3000, 2000, 500, and 1000 kg-m². Shafts 1 and 2 are made of steel and have diameters 30 cm and 10 cm and lengths 2 cm and 1.0 m, respectively. Find the natural frequencies of the system.



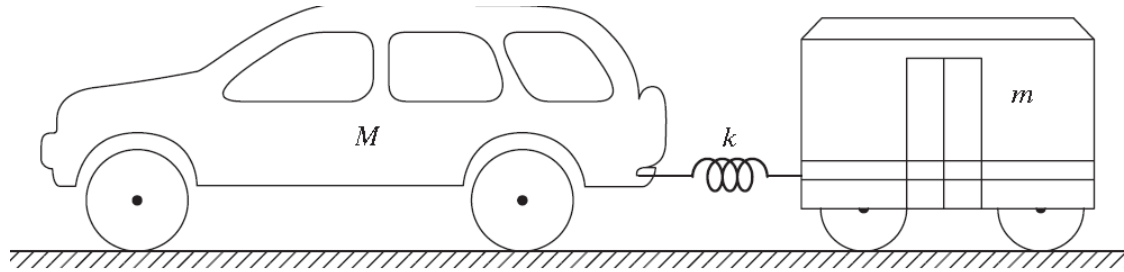
Exercise 12

A hot-air balloon of mass m is used to lift a load, Mg , by means of 12 equally spaced elastic ropes, each of stiffness k (see Fig.). Find the natural frequencies of vibration of the balloon in vertical direction. State the assumptions made in your solution and discuss their validity.



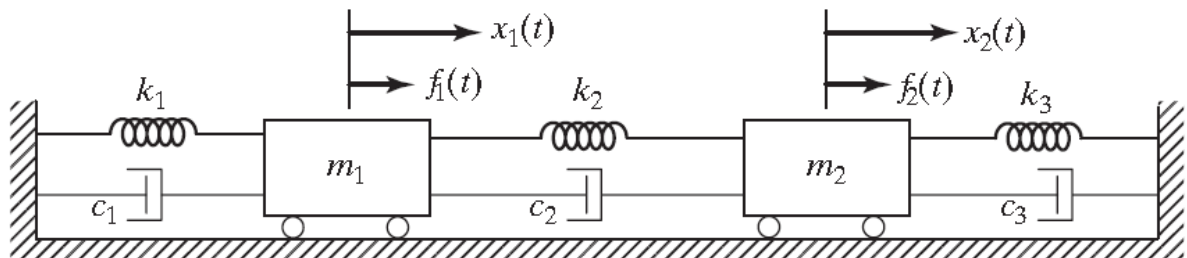
Exercise 13

The Figure shows a 3000-lb car connected to a 2000-lb trailer by a flexible hitch having a stiffness of 1000 lb/in. Assuming that both the car and the trailer can move freely on the roadway, determine the natural frequencies and mode shapes of vibration of the system, then find the response of the car-trailer system, if the values of initial displacement and velocity are 6 in. and 0 in./sec for the car and -3 in. and 0 in./sec for the trailer.



Exercise 14

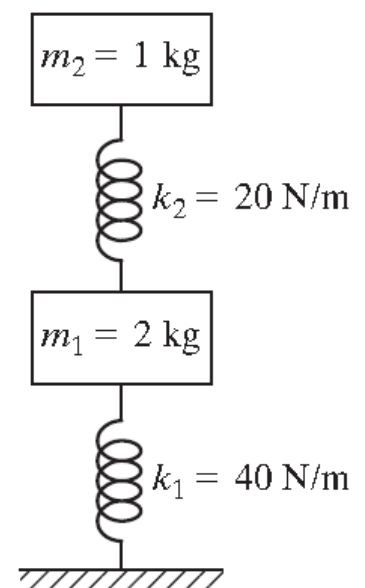
Find the free-vibration response of the system shown in the Figure using Laplace transform approach for the following data: $m_1 = 2$, $m_2 = 4$, $k_1 = 8$, $k_2 = 4$, $k_3 = 0$, $c_1 = 0$, $c_2 = 0$, $c_3 = 0$. Assume the initial conditions as $x_1(0) = 1$, $x_2(0) = 0$, and $\dot{x}_1(0) = \dot{x}_2(0) = 0$. Plot the responses $x_1(t)$ and $x_2(t)$.



Exercise 15

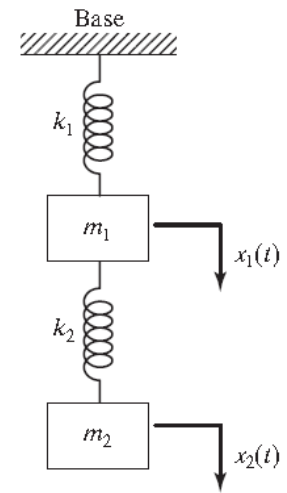
Find the response of the system shown in the following figure, with $m_1 = 2$, $m_2 = 1$, $k_1 = 40$ and $k_2 = 20$ for the following initial conditions using Laplace transform:

- A) $x_1(0) = 0.05$, $x_2(0) = 0.1$, $\dot{x}_1(0) = 0$, $\dot{x}_2(0) = 0$
- B) $x_1(0) = 0.1$, $x_2(0) = -0.05$, $\dot{x}_1(0) = 0$, $\dot{x}_2(0) = 0$



Exercise 16

For the system shown solve the following parts separately:



A)

Find the natural frequencies and mode shapes of the system shown in the Figure for $m_1 = m_2 = m$ and $k_1 = k_2 = k$.

B)

Find the natural frequencies and mode shapes of the system shown in the Figure for $m_1 = m_2 = 1$ kg, $k_1 = 2000$ N/m, and $k_2 = 6000$ N/m.

C)

Determine the initial conditions of the system shown in the Figure for which the system vibrates only at its lowest natural frequency for the following data: $k_1 = k$, $k_2 = 2k$, $m_1 = m$, $m_2 = 2m$.

D)

The system shown in the Figure is initially disturbed by holding the mass m_1 stationary and giving the mass m_2 a downward displacement of 0.1 m. Discuss the nature of the resulting motion of the system.

E)

Find the steady-state response of the system shown in the Figure by using the mechanical impedance method, when the mass m_1 is subjected to the force $F(t) = F_0 \sin \omega t$ in the direction of $x_1(t)$.

F)

Find the steady-state response of the system shown in the Figure when the base is subjected to a displacement $y(t) = Y_0 \cos \omega t$.

G)

The mass m_1 of the two-degree-of-freedom system shown in the Figure is subjected to a force $F_0 \cos \omega t$. Assuming that the surrounding air damping is equivalent to $c = 200$ N-s/m, find the steady-state response of the two masses. Assume $m_1 = m_2 = 1$ kg, $k_1 = k_2 = 500$ N/m, and $\omega = 1$ rad/s.